Homework 6 Solution

Chapter 5

Problem 5.2 When a particle with charge q and mass m is introduced into a medium with a uniform field **B** such that the initial velocity of the particle **u** is perpendicular to **B**(Fig. P5.2), the magnetic force exerted on the particle causes it to move in a circle of radius a. By equating $\mathbf{F}_m$ to the centripetal force on the particle, determine a in terms of q, m, u, and **B**.

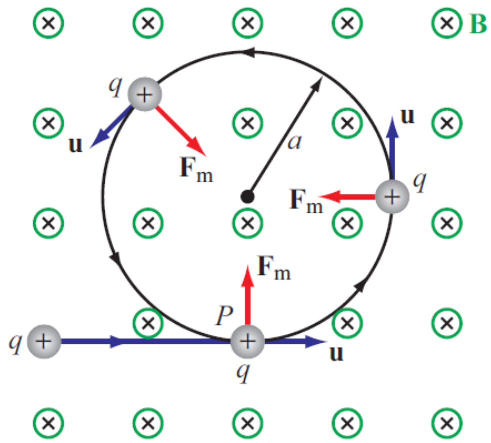

Figure P5.2: Particle of charge q projected with velocity **u** into a medium with a uniform field **B** perpendicular to **u** (Problem 5.2).

Solution: The centripetal force acting on the particle is given by $F_c = mu^2/a$. Equating F_c to F_m given by Eq. (5.4), we have $mu^2/a = quB\sin\theta$. Since the magnetic field is perpendicular to the particle velocity, $\sin\theta = 1$. Hence, $a = mu/qB$.

Problem 5.4 The rectangular loop shown in Fig. P5.4 consists of 20 closely wrapped turns and is hinged along the z-axis. The plane of the loop makes an angle of 30° with the y-axis, and the current in the windings is 0.5 A. What is the magnitude of the torque exerted on the loop in the presence of a uniform field $\mathbf{B} = \hat{\mathbf{y}}2.4$ T? When viewed from above, is the expected direction of rotation clockwise or counterclockwise?

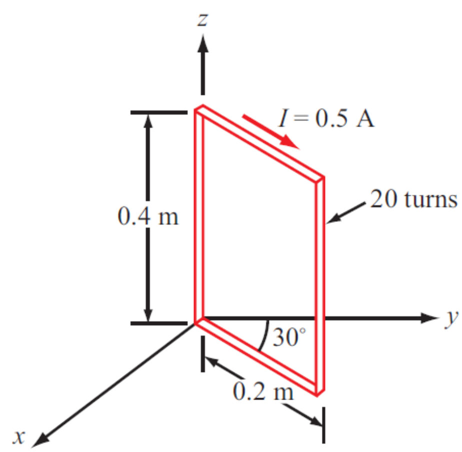

Figure P5.4: Hinged rectangular loop of Problem 5.4.

Solution: The magnetic torque on a loop is given by $\mathbf{T} = \mathbf{m} \times \mathbf{B}$ (Eq. (5.20)), where $\mathbf{m} = \hat{\mathbf{n}}NIA$ (Eq. (5.19)). For this problem, it is given that $I = 0.5$ A, $N = 20$ turns, and $A = 0.2 \text{ m} \times 0.4 \text{ m} = 0.08 \text{ m}^2$. From the figure, $\hat{\mathbf{n}} = -\hat{\mathbf{x}}\cos 30° + \hat{\mathbf{y}}\sin 30°$. Therefore, $\mathbf{m} = \hat{\mathbf{n}}0.8$ $(\text{A} \cdot \text{m}^2)$ and $\mathbf{T} = \hat{\mathbf{n}}0.8$ $(\text{A} \cdot \text{m}^2) \times \hat{\mathbf{y}}2.4 \text{ T} = -\hat{\mathbf{z}}1.66$ (N·m). As the torque is negative, the direction of rotation is clockwise, looking from above.

Problem 5.12 Two infinitely long, parallel wires are carrying 6-A currents in opposite directions. Determine the magnetic flux density at point P in Fig. P5.12.

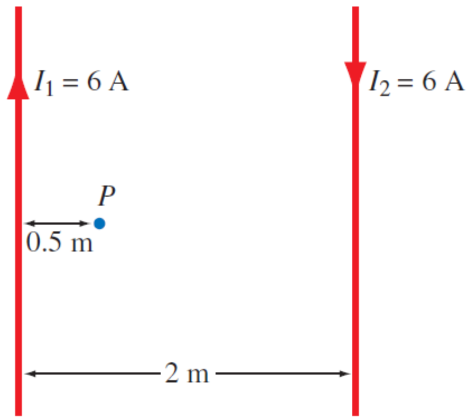

Figure P5.12: Arrangement for Problem 5.12.

Solution:

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I_1}{2\pi(0.5)} + \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I_2}{2\pi(1.5)} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0}{\pi}(6+2) = \hat{\boldsymbol{\phi}}\,\frac{8\mu_0}{\pi} \quad \text{(T)}.$$

5.15 A circular loop of radius a carrying current I_1 is located in the x–y plane as shown in Fig. P5.15. In addition, an infinitely long wire carrying current I_2 in a direction parallel with the z-axis is located at $y = y_0$.

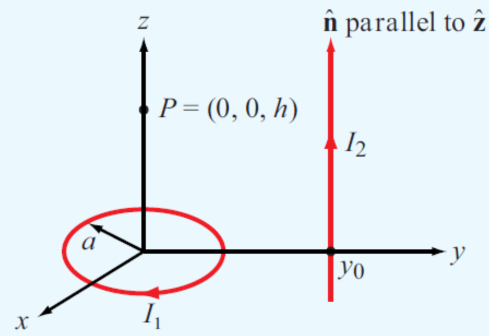

Figure P5.15 Problem 5.15.

(a) Determine **H** at $P = (0,0,h)$.

(b) Evaluate **H** for $a = 3$ cm, $y_0 = 10$ cm, $h = 4$ cm, $I_1 = 10$ A, and $I_2 = 20$ A.

Solution:

(a) The magnetic field at $P = (0,0,h)$ is composed of $\mathbf{H}_1$ due to the loop and $\mathbf{H}_2$ due to the wire:

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2.$$

Eq. (5.34) applies to a current in CCW direction, but in the loop of Fig. P5.15, the current is CW. Hence, with $z = h$ and adding a minus sign, we have

$$\mathbf{H}_1 = \hat{\mathbf{z}}\frac{-I_1 a^2}{2(a^2+h^2)^{3/2}} \qquad \text{(A/m)}.$$

From (5.30), the field due to the wire at a distance $r = y_0$ is

$$\mathbf{H}_2 = \hat{\boldsymbol{\phi}}\frac{I_2}{2\pi y_0}$$

where $\hat{\boldsymbol{\phi}}$ is defined with respect to the coordinate system of the wire. Point P is located at an angel $\phi = -90^\circ$ with respect to the wire coordinates. From Table 3-2,

$$\begin{aligned}\hat{\boldsymbol{\phi}} &= -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi \\ &= \hat{\mathbf{x}} \qquad (\text{at } \phi = -90^\circ).\end{aligned}$$

Hence,

$$\mathbf{H} = \hat{\mathbf{z}}\,\frac{-I_1 a^2}{2(a^2+h^2)^{3/2}} + \hat{\mathbf{x}}\,\frac{I_2}{2\pi y_0} \qquad \text{(A/m)}.$$

(b)

$$\mathbf{H} = -\hat{\mathbf{z}}\,36 + \hat{\mathbf{x}}\,31.83 \qquad \text{(A/m)}.$$

Problem 5.21 Current I flows along the positive z-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has radius a, and the inner and outer radii of the outer conductor are b and c, respectively.

(a) Determine the magnetic field in each of the following regions: $0 \le r \le a$, $a \le r \le b$, $b \le r \le c$, and $r \ge c$.

(b) Plot the magnitude of $\mathbf{H}$ as a function of r over the range from $r = 0$ to $r = 10$ cm, given that $I = 10$ A, $a = 2$ cm, $b = 4$ cm, and $c = 5$ cm.

Solution:

(a) Following the solution to Example 5-5, the magnetic field in the region $r < a$,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{rI}{2\pi a^2},$$

and in the region $a < r < b$,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}.$$

The total area of the outer conductor is $A = \pi(c^2 - b^2)$ and the fraction of the area of the outer conductor enclosed by a circular contour centered at $r = 0$ in the region $b < r < c$ is

$$\frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} = \frac{r^2 - b^2}{c^2 - b^2}.$$

The total current enclosed by a contour of radius r is therefore

$$I_{\text{enclosed}} = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = I\frac{c^2 - r^2}{c^2 - b^2},$$

and the resulting magnetic field is

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{I_{\text{enclosed}}}{2\pi r} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right).$$

For $r > c$, the total enclosed current is zero: the total current flowing on the inner conductor is equal to the total current flowing on the outer conductor, but they are flowing in opposite directions. Therefore, $\mathbf{H} = 0$.

(b) See Fig. P5.21.

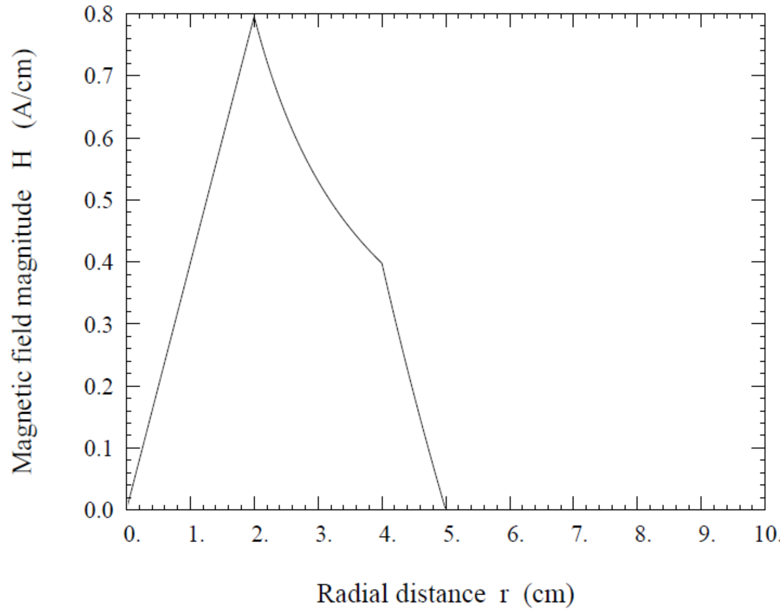

Figure P5.21: Problem 5.21.

Problem 5.28 A uniform current density given by

$$\mathbf{J} = \hat{\mathbf{z}} J_0 \quad (\text{A/m}^2)$$

gives rise to a vector magnetic potential

$$\mathbf{A} = -\hat{\mathbf{z}} \frac{\mu_0 J_0}{4}(x^2 + y^2) \quad (\text{Wb/m})$$

(a) Apply the vector Poisson's equation to confirm the above statement.

(b) Use the expression for **A** to find **H**.

(c) Use the expression for **J** in conjunction with Ampère's law to find **H**. Compare your result with that obtained in part (b).

Solution:

(a)

$$\begin{aligned}
\nabla^2 \mathbf{A} = \hat{\mathbf{x}} \nabla^2 A_x + \hat{\mathbf{y}} \nabla^2 A_y + \hat{\mathbf{z}} \nabla^2 A_z &= \hat{\mathbf{z}} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \left[-\mu_0 \frac{J_0}{4} (x^2 + y^2) \right] \\
&= -\hat{\mathbf{z}} \mu_0 \frac{J_0}{4} (2 + 2) = -\hat{\mathbf{z}} \mu_0 J_0.
\end{aligned}$$

Hence, $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$ is verified.

(b)

$$\begin{aligned}
\mathbf{H} = \frac{1}{\mu_0} \nabla \times \mathbf{A} &= \frac{1}{\mu_0} \left[\hat{\mathbf{x}} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{\mathbf{y}} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{\mathbf{z}} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \right] \\
&= \frac{1}{\mu_0} \left(\hat{\mathbf{x}} \frac{\partial A_z}{\partial y} - \hat{\mathbf{y}} \frac{\partial A_z}{\partial x} \right) \\
&= \frac{1}{\mu_0} \left[\hat{\mathbf{x}} \frac{\partial}{\partial y} \left(-\mu_0 \frac{J_0}{4} (x^2 + y^2) \right) - \hat{\mathbf{y}} \frac{\partial}{\partial x} \left(-\mu_0 \frac{J_0}{4} (x^2 + y^2) \right) \right] \\
&= -\hat{\mathbf{x}} \frac{J_0 y}{2} + \hat{\mathbf{y}} \frac{J_0 x}{2} \quad (\text{A/m}).
\end{aligned}$$

(c)

$$\begin{aligned}
\oint_C \mathbf{H} \cdot d\mathbf{l} = I &= \int_S \mathbf{J} \cdot d\mathbf{s}, \\
\hat{\boldsymbol{\phi}} H_\phi \cdot \hat{\boldsymbol{\phi}} 2\pi r &= J_0 \cdot \pi r^2, \\
\mathbf{H} = \hat{\boldsymbol{\phi}} H_\phi &= \hat{\boldsymbol{\phi}} J_0 \frac{r}{2}.
\end{aligned}$$

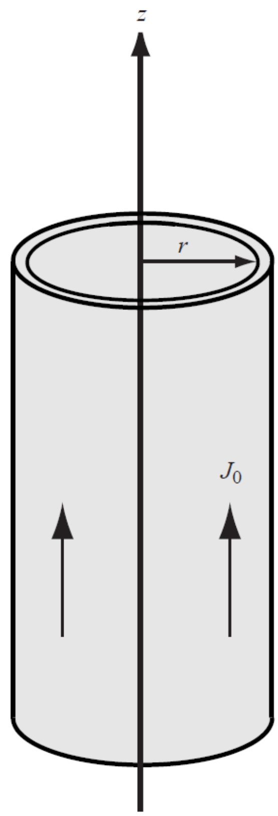

Figure P5.28: Current cylinder of Problem 5.28.

We need to convert the expression from cylindrical to Cartesian coordinates. From Table 3-2,

$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi = -\hat{\mathbf{x}}\frac{y}{\sqrt{x^2+y^2}} + \hat{\mathbf{y}}\frac{x}{\sqrt{x^2+y^2}}\,,$$

$$r = \sqrt{x^2+y^2}\,.$$

Hence

$$\mathbf{H} = \left(-\hat{\mathbf{x}}\frac{y}{\sqrt{x^2+y^2}} + \hat{\mathbf{y}}\frac{x}{\sqrt{x^2+y^2}}\right)\cdot\frac{J_0}{2}\sqrt{x^2+y^2} = -\hat{\mathbf{x}}\frac{yJ_0}{2} + \hat{\mathbf{y}}\frac{xJ_0}{2}\,,$$

which is identical with the result of part (b).

Problem 5.30 In the model of the hydrogen atom proposed by Bohr in 1913, the electron moves around the nucleus at a speed of 2×10^6 m/s in a circular orbit of radius 5×10^{-11} m. What is the magnitude of the magnetic moment generated by the electron's motion?

Solution: From Eq. (5.69), the magnitude of the orbital magnetic moment of an electron is

$$|m_0| = \left|-\tfrac{1}{2}eur\right| = \tfrac{1}{2} \times 1.6 \times 10^{-19} \times 2 \times 10^6 \times 5 \times 10^{-11} = 8 \times 10^{-24} \quad (\text{A}\cdot\text{m}^2).$$

Problem 5.32 The x–y plane separates two magnetic media with magnetic permeabilities μ_1 and μ_2 (Fig. P5.32). If there is no surface current at the interface and the magnetic field in medium 1 is

$$\mathbf{H}_1 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{y}} H_{1y} + \hat{\mathbf{z}} H_{1z}$$

find:

(a) $\mathbf{H}_2$

(b) θ_1 and θ_2

(c) Evaluate $\mathbf{H}_2$, θ_1, and θ_2 for $H_{1x} = 2$ (A/m), $H_{1y} = 0$, $H_{1z} = 4$ (A/m), $\mu_1 = \mu_0$, and $\mu_2 = 4\mu_0$

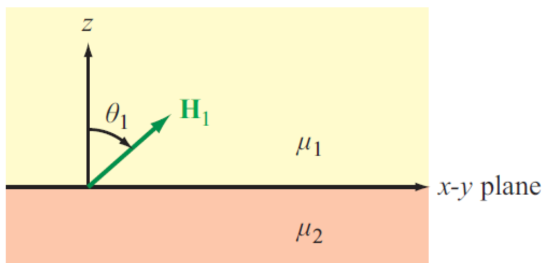

Figure P5.32: Adjacent magnetic media (Problem 5.32).

Solution:

(a) From (5.80),

$$\mu_1 H_{1n} = \mu_2 H_{2n},$$

and in the absence of surface currents at the interface, (5.85) states

$$H_{1t} = H_{2t}.$$

In this case, $H_{1z} = H_{1n}$, and H_{1x} and H_{1y} are tangential fields. Hence,

$$\mu_1 H_{1z} = \mu_2 H_{2z},$$
$$H_{1x} = H_{2x},$$
$$H_{1y} = H_{2y},$$

and

$$\mathbf{H}_2 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{y}} H_{1y} + \hat{\mathbf{z}} \frac{\mu_1}{\mu_2} H_{1z}.$$

(b)

$$H_{1t} = \sqrt{H_{1x}^2 + H_{1y}^2},$$

$$\tan\theta_1 = \frac{H_{1t}}{H_{1z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{H_{1z}},$$

$$\tan\theta_2 = \frac{H_{2t}}{H_{2z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{\frac{\mu_1}{\mu_2} H_{1z}} = \frac{\mu_2}{\mu_1} \tan\theta_1 .$$

(c)

$$\mathbf{H}_2 = \hat{\mathbf{x}}\,2 + \hat{\mathbf{z}}\,\frac{1}{4} \cdot 4 = \hat{\mathbf{x}}\,2 + \hat{\mathbf{z}} \quad \text{(A/m)},$$

$$\theta_1 = \tan^{-1}\left(\frac{2}{4}\right) = 26.56^\circ,$$

$$\theta_2 = \tan^{-1}\left(\frac{2}{1}\right) = 63.44^\circ.$$

Problem 5.38 A solenoid with a length of 20 cm and a radius of 5 cm consists of 400 turns and carries a current of 12 A. If $z = 0$ represents the midpoint of the solenoid, generate a plot for $|\mathbf{H}(z)|$ as a function of z along the axis of the solenoid for the range -20 cm $\leq z \leq 20$ cm in 1-cm steps.

Solution:

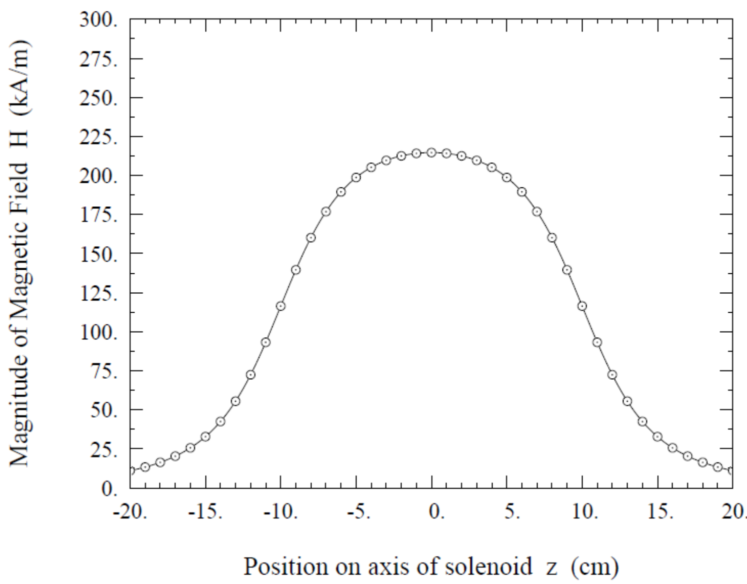

Figure P5.38: Problem 5.38.

Let the length of the solenoid be $l = 20$ cm. From Eq. (5.88a) and Eq. (5.88b), $z = a\tan\theta$ and $a^2 + t^2 = a^2\sec^2\theta$, which implies that $z/\sqrt{z^2+a^2} = \sin\theta$. Generalizing this to an arbitrary observation point z' on the axis of the solenoid, $(z-z')/\sqrt{(z-z')^2+a^2} = \sin\theta$. Using this in Eq. (5.89),

$$\begin{aligned}
\mathbf{H}(0,0,z') = \frac{\mathbf{B}}{\mu} &= \hat{\mathbf{z}}\frac{nI}{2}(\sin\theta_2 - \sin\theta_1) \\
&= \hat{\mathbf{z}}\frac{nI}{2}\left(\frac{l/2-z'}{\sqrt{(l/2-z')^2+a^2}} - \frac{-l/2-z'}{\sqrt{(-l/2-z')^2+a^2}}\right) \\
&= \hat{\mathbf{z}}\frac{nI}{2}\left(\frac{l/2-z'}{\sqrt{(l/2-z')^2+a^2}} + \frac{l/2+z'}{\sqrt{(l/2+z')^2+a^2}}\right) \quad \text{(A/m)}.
\end{aligned}$$

A plot of the magnitude of this function of z' with $a = 5$ cm, $n = 400$ turns$/20$ cm $=$ $20,000$ turns/m, and $I = 12$ A appears in Fig. P5.38.